A robust design of digital pulse compression filter for radar application based on MMSE approach

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1. Introduction

Aim of the paper is to describe two adjustments to the "conventional" Minimum Mean Square Error (MMSE) algorithm applied to the digital compression of codified pulses. The use of linear or non linear frequency modulated (LFM or NLFM) pulses is common in the radar field to achieve good target resolution while limiting the transmitted peak power. The adjustments to the MMSE derive from the experience gained by processing the real data acquired from a number of radar systems; the main benefits of adjusted MMSE are the capability of synthesizing digital compression filters showing (i) very low sidelobes and (ii) low degradation in presence of thermal noise. The thermal noise, usually called system noise in radar jargon, is emulated via random samples with 0 mean value, parametric standard deviation and gaussian amplitude distribution.

The paper is organized as follows: next section 2 presents, with the help of a pseudo-code using Matlab® notations, the conventional and the modified MMSE procedure with application to the digital radar pulse compression. Section 3 relates to the results achieved via Monte Carlo simulation; while section 4 contains an example of real data digital pulse compression pertinent to a three-dimensional (range, azimuth and elevation) radar system. At the end, sections 5 and 6 reports conclusions and references.

2. MMSE procedure

The conventional formulation of the MMSE procedure applied to the digital pulse compression is the following:

<u>Step 1</u>.
A "goal function" has to be defined which represented the wanted shape of the compressed pulse. Using a pseudo-code representation of the MMSE algorithm, the following operation have to be performed (note that the sentences preceded by the symbol % have to be considered as comments).

% Definition of goal function
LD = LF+LS-1; *%output dimension (LF :dimension of the MMSE filter, LS: code dimension).*
df = chebwin(fix(LD), R_dB); *Chebychef function with a sidelobe level equal to –R_dB and dimension equal to LD.*
d = fftshift(fft(df, LD)); *shift to have the maximum of the function in correspondence of the middle point of the vector.*
d = abs(d); *% final definition of goal function*

<u>Step 2</u>
Computation of the autocorrelation function of the coded pulse ("s" is the variable containing the input signal) .

oldsig=s; *%download in oldsig of the s signal without "zero padding".*
s = [s; zeros(floor((LF-LS)),1)]; *%zero padding of the input signal s up to the desired MMSE filter length.*
Pss = (xcorr(s)); *% Computation of the autocorrelation matrix of the signal s.*

%Selection and dowload in the R matrix of the autocorrelation matrix of dimension LFxLF where LF is the MMSE filter length

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for i = 1:LF
    R(i,:)=Pss(floor((2*LF-1)/2+1)-(i-1):length(Pss)-(i-1)).';
end
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Step 3

Computation of the MMSE filter. The theory requires to compute the correlation of the input signal not "zero padded" with the goal function; then the filter is computed via the product of the inverse of the input signal autocorrelation matrix with the correlation vector above defined. The corresponding pseudo-code is:

psdtot= xcorr(oldsig,desid); *%correlation between the input signal and the goal function*
% Computation of the MMSE filter which is downloaded in the vector h
[kk mm]=max(abs(psdtot));
indcentr=mm;
h_tmp = inv(R)*psdtot(indcentr-LF/2+1:indcentr+LF/2);
h = conj(h_tmp);

2.1. Modification of the goal function

The first modification which is mandatory to obtain very low sidelobes is pertinent to the goal function. The following figure 1 presents an example of a goal function of length equal to 250 samples and related to a Fourier transform of a Chebycheff function with –70 dB sidelobes value with respect to the peak of the compressed pulse. The figure presents the squared amplitude in dB of the Chebycheff versus the samples order. Selecting this type of goal function implies severe numerical problems because the MMSE filter tries to follow also the very low sidelobes (around –100 dB in figure 1); tracking the very low sidelobes degrades the values of the achieved sidelobes in different "zones" of the compressed pulses where the lobes increase up to –40 dB instead of the –70 dB required. Note that this result has been found using a MMSE filter length equal to three times the code length, which is the recommended dimension of the MMSE compression filter.

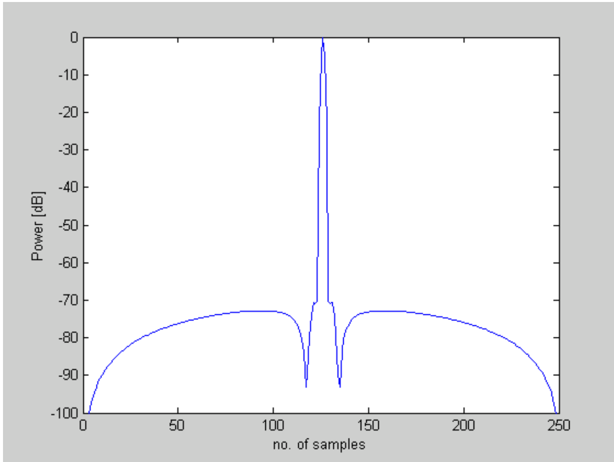

Figure 1: Fourier transform of the Chebycheff function with –70 dB sidelobe level.

It is needed to modify the goal function trying to remove the very low sidelobes zones. A number of different attempts have been performed and the more robust technique found is the following: if a 250 points goal function is required, a shorter function has to be defined (as an example 125 points instead of 250); then the goal function is zero padded up to the desired length (125 zeros are needed in the example under analysis). The Fourier transform of the zero padded function is finally performed. The cost of the Chebycheff modification (i.e. shortening) is the broadening of the main lobe and the loss in the peak gain of the compressed pulse.

In the study case under analysis it has been found a very limited broadening of the beam a loss of gain lower than 0.5 dB which is still acceptable. The new goal function is presented in figure 2.

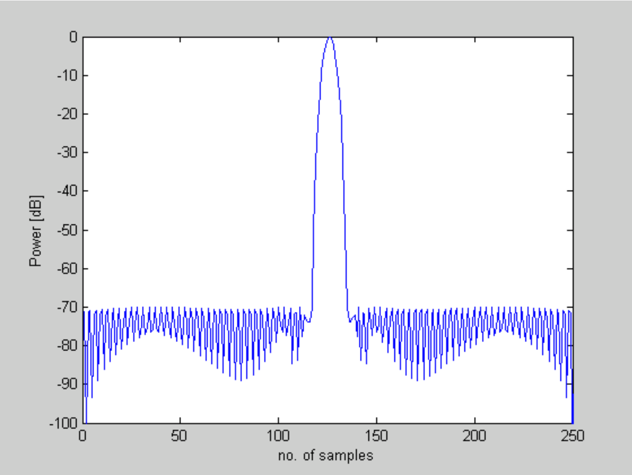

Figure 2: Fourier transform of the zero padded Chebycheff function with –70 dB sidelobe level

The benefit of the zero-padded goal function is evident in next figure 3, which reports a NLFM code compressed with two MMSE filters: the first one (red curve) is derived using a "not zero-padded" goal function, the second (blue curve) is extracted from as "zero-padded" goal function. The NLFM code has the following characteristics: time duration 16 μsec., bandwidth=2.5 MHz, sampling frequency 5 MHz.

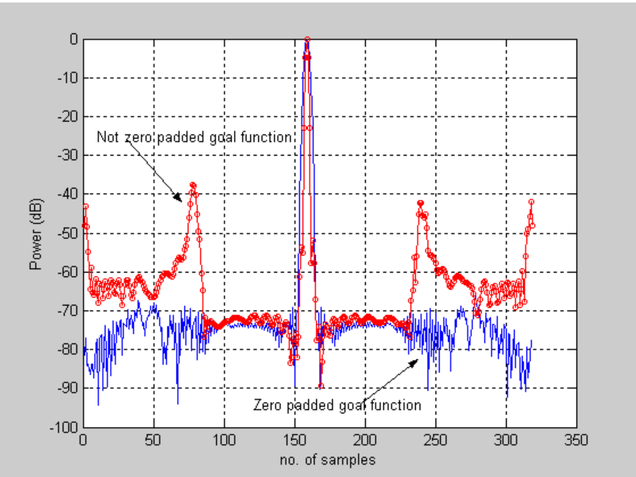

Figure 3. MMSE compressed pulse

2.2. *Modification of filter computation procedure*

The modified procedure starts from the following consideration: in case of absence of thermal noise, the autocorrelation of a LFM/NLFM coded signal is real, i.e. the imaginary part of the autocorrelation is equal to 0. In this case, the autocorrelation matrix can be computed using two different numerical operations: (i) compute the autocorrelation via the *xcorr(s)* Matlab® instruction or (ii) computing the convolution of the signal itself with its conjugated replica. The corresponding Matlab® instruction is *conv (s,s')*. The two procedures differ for the rotation on the conjugated replica and in case of symmetric input signal, like a LFM code, the rotation does not change the autocorrelation function..

Let us suppose of adding thermal noise samples (*n*) to the useful signal *s*. The new signal *s+n* is not symmetric. Using a numerical example, it is possible to show that the matrix computed with procedures (i) and (ii) above described are quite similar, but:

1) using the procedure (ii) the output matrix is real, and
2) the degradation of the sidelobes level caused by the thermal noise is very limited.

The result described in point 2) above has certain impact in practical cases; in fact the MMSE procedure is used to achieve very low sidelobes of the compressed signal in presence of mismatch of the radar receiving channel. To reach the goal, a replica of the transmitted signal is entered in the radio-frequency section of the radar and the signal samples are recorded after the analog-to-digital converter. The acquired signal passed the radio and intermediate frequency sections of the radar receiver chain thus it contains the non-linearity that degrades the ideal response of the digital pulse compression filter. The MMSE approach is then employed to synthesize the compression filter that compensates for the mismatch of the receiver chain; in the full paper it will be shown that the procedure (ii) gives better results with respect to the procedure (i) also in this practical application.

2.2.1. Numerical example

Given the numerical sequence: **a**=[1+i*1 3+i*3 1+i*1], its autocorrelation function is: *xcorr(a)=(conv(a,a'))*= 2 12 22 12 2.

Consider now the vector **a_n** derived from the vector a plus four samples of thermal noise as defined in section 1: **a_n**=[1.0026 + 0.9987i 3.0121 + 2.9873i 0.9973 + 0.9834i].

The autocorrelation function of **a_n** is: *xcorr(a_n)*= [1.9819 + 0.0100i 11.9448 + 0.0301i 21.9610 11.9448 - 0.0301i 1.9819 - 0.0100i].

The vector computer by the *conv(a_n,a_n')* is=: 2.0026 12.0067 21.9607 11.8829 1.9615 which is real and it appears quite similar to the autocorrelation function of **a**.

The convolution of the signal **a_n** with its conjugated replica permits to eliminate the imaginary terms of the autocorrelation function that are mainly caused by the added noise samples; next section 3 presents the benefits of the use of the signal convolution in lieu of the autocorrelation function with reference to the same study case described in section 2.1.

3. Achieved results

In absence of thermal noise, both procedures (i) and (ii) gives same MMSE compression filter that, when applied at the NLFM signal defined in section 2.1, return the compressed pulse presented in figure 4.

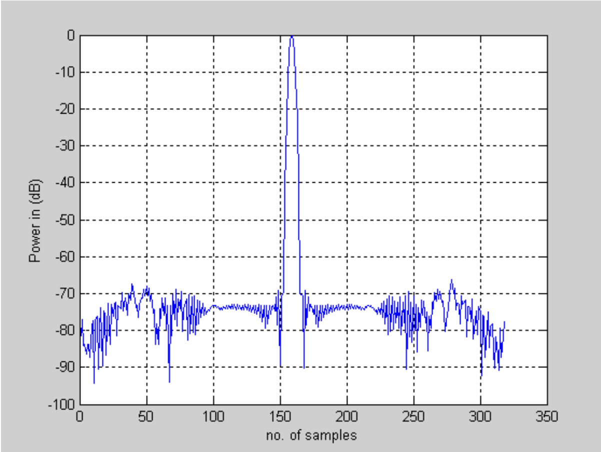

Figure 4: compressed NLFM code in absence of thermal noise

Next figure 5 depicts two curves relevant to the same study case above described but with a signal-to-thermal noise power ratio (SNR) equal to 10 dB. Red curve has been obtained designing the MMSE filter with procedure (i) and the blue curve is derived from the filter designed with the procedure (ii). The advantage of procedure (ii) is clear.

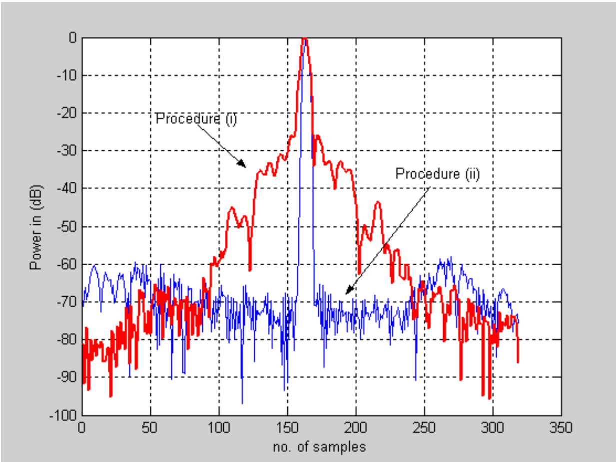

Figure 5. Compressed signal with MMSE filters and SNR=10dB

4. Application of MMSE to real data

This section relates to the results achieved in applying the modified MMSE procedure to the chirp recorded from a typical 3D radar produced by the Company. The chirp has the following characteristics: length>20 μs, sampling time<0.5 μs, linear increasing frequency modulation, SNR values>50 dB. Next figure 6 depicts two curves pertinent to the real chirp above described. Red curve has been obtained designing the MMSE filter with procedure (i) and the blue curve is derived from the filter designed with the procedure (ii). The advantage of procedure (ii) is clear also for the real data case.

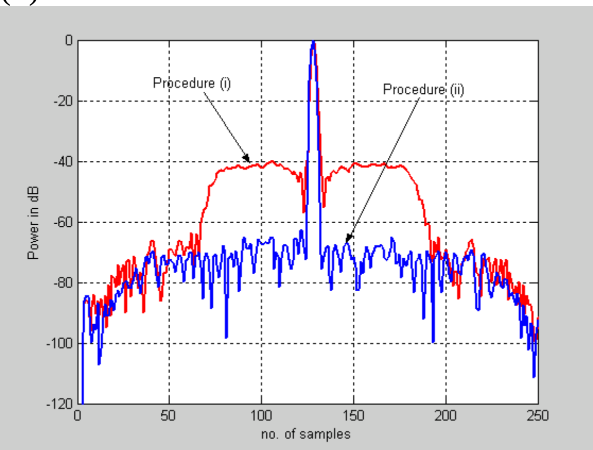

Figure 6. Compressed real chirp with MMSE filters and SNR>50dB

5. Conclusions

A couple of adjustments derived from the experience gained in the application of the MMSE technique to the design of digital pulse compression filter have been described and simulation examples have been shown. The full paper will contain example of the benefit of the adjustments to the real data acquired from one of the radar produced by the Company.

6. References

[1]A. Di Vito, A. Farina, F.A. Studer "Design of low sidelobe filters with MMSE technique" Memo/fr/prog/as/216/83, 25-8-1983, Internal AMS Memo

[2] S. Haykin, "Adaptive Filter Theory", fourth edition, Prentice Hall,2002.

[3] A. Farina:" Antenna Based Signal Processing Techniques for Radar Systems", Artech House, 1992.

[4] A. Farina, G. Golino, L. Timmoneri, "Comparison between LS and TLS in adaptive processing for radar systems". Proc. of IEE Radar, Sonar and Navigation, 2003, no. 1, February 2003, pp. 2-6.