Classification and Launch-Impact Point Prediction of Ballistic Target via Multiple Model Maximum Likelihood Estimator (MM-MLE)

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Abstract. **The paper deals with the problems of (i) launch and impact point prediction (LPP, IPP) of Ballistic Targets (BT) and (ii) BT classification by processing measurements acquired either by 3D surveillance or multifunctional phased-array radars . It is assumed that the radar acquires a limited number of measurements (plots) that do not encompass the whole target trajectory; thus the established target track has to be extrapolated ahead in time in order to predict the coordinates of the impact point. A procedure based on Multiple Model Maximum Likelihood Estimator (MM-MLE) has been conceived and tested using a Monte Carlo simulation approach; the parameters selected for testing are the probability of BT correct classification (Pcc), the IPP and the LPP. The new procedure is compared with the estimator described in [4].**

I. INTRODUCTION

This paper will present a Classification and Launch-Impact Point Prediction procedure exploiting a-priori rough indications stored in a BT models database.
The following three ingredients are the basis for the BT classification and the consequent IPP:

- a detailed model of the BT dynamic and kinematics characteristics (see [1], [2], [3], [6]),
- a BT Database (DB) with a number of information concerning the expected BTs,
- a Multiple Model Maximum Likelihood Estimator (MM-MLE) for finding the actual BT among those listed in the DB and performing the IPP.

Once that the BT model is built, it is possible to conceive proper ballistic target tracking filters and estimators of the launch and impact points. It has been shown [3] that, if an a-priori knowledge of the BT physical parameters is available (i.e. the ballistic coefficient), then the MLE batch estimator gives remarkable better performance with respect to the recursive filters. The main problem in the application of the impact point estimators is the loss of knowledge of the BT specific parameters. For the recursive filters and estimators and for the BT in cruise and re-entry phases, the problem has been solved ([3] and [5]) with the application of a multiple model approach. The recursive filters have been integrated in a Interacting Multiple Model (IMM) tracking architecture for the simultaneous estimate of BT ballistic coefficient and of the target velocity and position.

The purposes of the original MM-MLE procedure described in this paper are:

(i) take advantage of the boost phase plots acquired by the surveillance system to reduce the time required for a robust LPP and IPP,

(ii) avoid the implementation of recursive filters estimating the BT unknown parameters via the use of a database containing kinematics information and rough indication of BT technical data; note that few parameters (e,g. burn-out time) are critical for a successful IP prediction, thus a dedicated procedure for accurately estimating them has been implemented (see section III for further details);

(iii) classify the BT, by selecting the correct model among those listed in the DB.

II. MODELS OF BT KINEMATIC AND RADAR MEASURES

II.1 BT kinematics

Three main forces affect the BT motion: thrust, drag and gravity [6].

The thrust acts along the axis of the rocket and therefore it is aligned with its velocity vector:

$$\vec{a}_{th} = \frac{T}{M} \cdot \vec{u}_V \qquad (1.)$$

The thrust components are obtained by multiplying the total thrust acceleration by the components of the unitary vector of the velocity:

$$\vec{a}_{th} \equiv \begin{bmatrix} a_{th_x} \\ a_{th_y} \\ a_{th_z} \end{bmatrix} = \frac{T}{M(t) \cdot \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}} \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \qquad (2.)$$

where

$$T = g_0 \cdot I_{sp} \cdot \frac{M_{fuel}}{t_{BO}} \quad (3.)$$

being:

1 g_0 is the acceleration due to gravity at the sea level (9.81 m/s^2);
2 I_{sp} (in s) is the specific impulse that is a peculiar property of the propellant system, being the thrust produced per unit mass rate. Its value varies with the operating conditions and design of the rocket engine (e.g. the combustion chamber pressure). Typical values for I_{sp} range in the interval: 200 to 300 sec;
3 M_{fuel} is the mass of the propellant;
4 t_{BO} is time of burn-out (s).

The instantaneous mass of the missile M(t) is expressed in a form useful for trajectory calculations in the following way:

$$\mathrm{M(t)} = \left(\mathrm{M_{body}} + \mathrm{M_{fuel}}\right) \cdot \left(1 - \mathrm{mf} \cdot \frac{\mathrm{t}}{\mathrm{t_{BO}}}\right) \quad (4.)$$

being M_{body} (kg) the empty missile mass, M_{fuel} (kg) the initial mass of the propellant and mf is the fuel mass fraction given by:

$$\mathrm{mf} = \frac{\mathrm{M_{fuel}}}{\mathrm{M_{body}} + \mathrm{M_{fuel}}} \quad (5.)$$

and represents a parameter of the propellant efficiency. We assume that the rocket has a single-stage, i.e. all the propellant is consumed during the burn time of the missile, t_{BO}.
The drag acceleration expression is [6]

$$\mathbf{a}_{drag} = \begin{bmatrix} a_{drag} \\ a_{drag} \\ a_{drag} \end{bmatrix} = -\frac{1}{2}\frac{\rho(z) \cdot g_0}{\beta}\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \quad (6.)$$

where β is the ballistic coefficient (N/m^2), $\rho(z)$ is the air density function of the height:

$$\rho(z) = 1.21907 \cdot e^{-z/9146.64} \quad (7.)$$

and $\dot{x}, \dot{y}, \dot{z}$ are the velocity components of the BT along the three axes of a Cartesian reference system. The gravity acceleration is considered constant, g_0=9.8$m/s^2$and directed along the z-axis.

II.2 Radar measurements

The measurements, collected by the radar for target tracking, are the range r, elevation ε and azimuth θ; the radar is located at: x_R=0, y_R=0, z_R=0 in all the numerical evaluations that will be done in the paper. The error standard deviations of these measurements are denoted as σ_r (for range), σ_θ (for elevation) and σ_ε (for azimuth). Since the BT kinematical model has been described in a Cartesian reference frame, the measurement equation is non linear:

$$\mathbf{z}_k = \mathbf{H}(\mathbf{x}_k) + \mathbf{v}_k \quad (8.)$$

where $\mathbf{x} = [x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z}]^T$ is the state vector and $\mathbf{v}_k$ is measurements noise (zero-mean white Gaussian noise). More precisely, the components of the function **H** for acquisition time are the following:

$$\begin{cases} R = h(\mathbf{x},k) = \sqrt{x(t_k)^2 + y(t_k)^2 + z(t_k)^2} \\ \varepsilon = f(\mathbf{x},k) = arcsin\dfrac{z(t_k)}{\sqrt{x(t_k)^2 + y(t_k)^2 + z(t_k)^2}} \\ \vartheta = l(\mathbf{x},k) = arctg\dfrac{x(t_k)}{y(t_k)} \end{cases} \quad (9.)$$

III. MM-MLE PROCEDURE

The proposed technique uses a Maximum Likelihood approach to the estimation of the Impact Point (IP) of the TBM. The aim is to find the set of initial conditions (for the ordinary differential equations describing the BT motion) which maximize the likelihood function achieved under the hypothesis of additive and Gaussian measurement error. The probability density function (pdf) of the error vector $\mathbf{e}$ of the radar measurements $R, \vartheta, \varepsilon$ (range, azimuth and elevation respectively) can be written as:

$$P(\mathbf{e}) = \frac{1}{\left[(\pi)^M |\mathbf{M}_C|\right]^{N/2}} \exp\left[-Tr\left\{\mathbf{M}_C^{-1} \sum_{k=1}^{N} \mathbf{e}_k \mathbf{e}_k^H\right\}\right] \quad (10.)$$

where $\mathbf{M}_C$ is the covariance matrix, M=3 is its dimension and Tr{**A**}and |**A**| stand for the trace and the determinant of the matrix **A** respectively.
Assuming the measurements errors statistically independent, $\mathbf{M}_C$ is a diagonal matrix:

$$\mathbf{M}_C = \begin{bmatrix} 2\sigma_R^2 & 0 & 0 \\ 0 & 2\sigma_\theta^2 & 0 \\ 0 & 0 & 2\sigma_\varepsilon^2 \end{bmatrix} \quad (11.)$$

Note that in this case the MLE reduces to the classical Least Square Estimation (LSE), that is the minimization of the exponent of the Gaussian pdf or equivalently of its logarithmic version.
The estimated set of initial conditions **p** can be written as:

$$\mathbf{p} = \arg\min_{\mathbf{p}} \lambda(\mathbf{x}(\mathbf{p})) \quad (12.)$$

where:

- $\mathbf{p} = \{x_{Launch}, y_{Launch}, Heading, Launch\ Angle\}$;
- $\mathbf{x} = [x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z}]^T$ is the state vector, obtained via numerical integration of the differential equations starting from the initial conditions **p**; the parameters needed in the equations for each BT model are stored in the DB;
- $\lambda(\mathbf{x}) = \sum_{k=1}^{N} \left\{ \frac{[R(\mathrm{k}) - h(\mathbf{x},k)]^2}{2\sigma_R^2} + \frac{[\theta(\mathrm{k}) - f(\mathbf{x},k)]^2}{2\sigma_\theta^2} + \frac{[\varepsilon(\mathrm{k}) - l(\mathbf{x},k)]^2}{2\sigma_\varepsilon^2} \right\}$ is the functional to be minimized in order to maximize the likelihood function;
- N is the number of radar measurements, k identifies the plot acquisition time.

Figure 1 depicts the flow-graph of the proposed MM-MLE classifier and estimator; it is mainly based on a number of MLEs running in parallel on a set of radar measurements; the input to the MLEs are the radar measurements and the database information. It must be noted that the measurements are a mixture of detections during boost and ballistic phases and they are all used for classification and LP & IP predictions. For each BT Model, the information which are contained in the Database in order to run the MM-MLE algorithm are the following:

- time dependent constraints on BT altitude (a.s.l.);
- time dependent constraints on BT velocity;
- time dependent constraints on BT acceleration;
- rough indication of drag coefficient;
- a range of admissible values for the t_{BO};[1];
- BT specific impulse;
- body mass;
- maximum value of fuel mass;

Each MLE receives:
(i) the above listed information for each one of the BT under analysis from the BT database, and
(ii) the actual detections from the radar.

[1] Multiple evaluations of eq. 11 will be performed over a set of t_{BO} values; the t_{BO} which provides the better result (in term of likelihood function) will be assigned to the BT for any further evaluation.

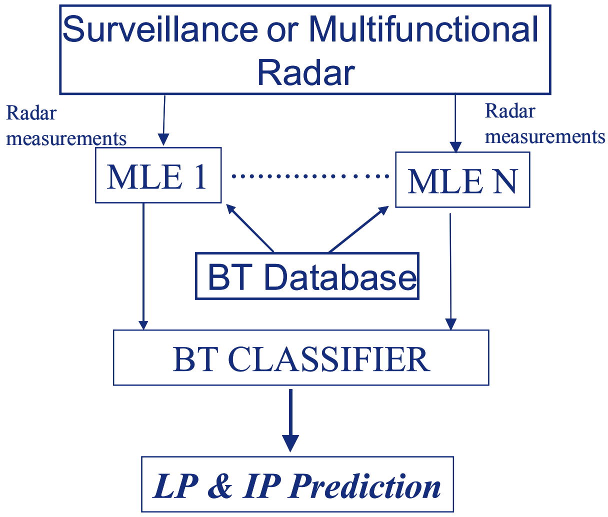

Figure 1. Flow graph of the MM-MLE approach

The aim of each MLE is to find the burn-out time (i.e. time duration of the BT boost phase) , the vertical and horizontal launch angles and the LP which maximize the likelihood function. All the MLEs send their own predictions to the BT classifier which selects the "true" BT model (i.e., the one showing the highest value of the likelihood function). Once that the BT parameters have been established, the IP prediction is performed via the BT equations of motion.

IV. RESULTS

The Monte Carlo approach is applied to determine the performance of the MM-MLE. In addition to the percentage of correct target classification and t_{BO} estimation, the accuracy (3σ values) in the prediction of the BT LP & IP is evaluated. The BT IP-LP are computed using the dynamic equations defined via the BT model starting from the output of the BT classifier. The numerical simulations have been performed over a set of trajectory characterized by the parameters reported in Table 1.

Table 1: BT parameters

	Beta (N/m^2)	TBO (s)	Mb (kg)	Mf (kg)	Isp (s)
BT A (Medium Range)	80000	74	300	2700	5400
BT B (Medium Range)	90000	60	350	2000	2800
BT C (Short Range)	120000	30	300	1800	1500
BT D (Short Range)	120000	30	330	2500	2300
BT E (Long Range)	60000	80	400	3000	6000
BT F (Long Range)	50000	100	450	4000	7000

The BT class is defined as follows: Short Range for BT having a range of 150-300 km, Medium Range for BT having a range of 400-700 km, Long Range for BT having a range of 800-1000 km.
For each trajectory the proposed MM-MLE procedure has been applied after 5 and 10 acquired radar measurements. In all the simulation, the first detection opportunity is assumed to subsist after 40 seconds from the BT launch.

Table 2 and Table 3 report the classification performances. The results show that the proposed method behaves correctly and, even after only 5 acquisitions, it provides a very high probability of correct classification. The classification errors reported in
Table 2 are present, as expected, among BTs belonging to the same class.

Table 2: Classification Performance after 5 plots

CLASSIFICATION %						
	BT A	BT B	BT C	BT D	BT E	BT F
BT A	93	7	0	0	0	0
BT B	6	94	0	0	0	0
BT C	0	0	89	11	0	0
BT D	0	0	9	91	0	0
BT E	0	0	0	0	87	13
BT F	0	0	0	0	8	92

Table 3: Classification Performance after 10 plots

CLASSIFICATION %						
	BT A	BT B	BT C	BT D	BT E	BT F
BT A	100	0	0	0	0	0
BT B	0	100	0	0	0	0
BT C	0	0	100	0	0	0
BT D	0	0	0	100	0	0
BT E	0	0	0	0	100	0
BT F	0	0	0	0	0	100

Table 4 and Table 5 show the results for the t_{BO} estimation. The central column represent the percentage of exact estimation, while side columns represent percentages of estimation errors of 1 or 2 seconds.
Via inspection of Table 1, it is clear that for BT C and BT D all the measurements are obtained during ballistic phase; for BT E and BT F only boost phase measurements are acquired in case of 5 plots, while we have both ballistic and boost phase measurements in case of 10 plots; for BT A and BT B we always have ballistic and boost phase measurements. These considerations help us to understand the results reported in Table 4 and Table 5; in fact, in order to have a satisfactory t_{BO} estimation, plots belonging both to boost phase and ballistic phase are needed. Therefore, the following considerations apply:

- for BT C and BT D the t_{BO} estimation is very inaccurate; nevertheless, having only ballistic plots to process, there is really no need to use the boost phase equation of motion for the IPP;
- for BT A and BT B, with only 5 plots we still have a reasonable chance to estimate burn-out time (around 40%), while we obtain good performances after 10 plots (75%);
- for BT E and BT F, after 5 plots the boost phase is still running; after 10 plots we obtain 80% of success percentage.

Table 4: t_{BO} estimation after 5 plots

BURN OUT TIME ESTIMATION %					
	TBO-2	TBO-1	TBO	TBO+1	TBO+2
BT A	9	20	39	21	11
BT B	7	23	40	21	9
BT C	21	20	20	19	20
BT D	19	22	23	19	17
BT E	19	21	22	18	20
BT F	19	20	23	17	21

Table 5: t_{BO} estimation after 10 plots

BURN OUT TIME ESTIMATION %					
	TBO-2	TBO-1	TBO	TBO+1	TBO+2
BT A	1	12	70	16	1
BT B	1	10	76	12	1
BT C	31	16	25	14	14
BT D	20	14	32	14	20
BT E	1	17	70	11	1
BT F	1	8	80	10	1

Finally, Table 6 and Table 7 show an overall performance indicator, that is the accuracy (in terms of major axis of uncertainty ellipse) of the IP and LP predictions. The first column (IP-std) represent the performance of a purely ballistic MLE as described in [3]. Figure 2 shows an example of simulation results. The dashed line refers to MLE described in [3], while the solid line refers to the newly proposed MM-MLE approach.
The results show that MM-MLE performs considerably better whenever there are boost phase measurements that the other MLE procedure is not able to process. For longer trajectories (thus having a longer boost phase) the accuracy improvement reach 50%. Moreover, since this method correctly manage the boost phase, it can provide IPPs and LPPs even when this phase is not ended yet (see Table 6), whereas the aforementioned MLE cannot be applied.

Table 6: IP-LP prediction accuracy after 5 plots

IP PREDICTION ACCURACY (m)				
	IP-std	IP	LP	%improv.
BT A	N.A.	27985	2103	N.A.
BT B	N.A.	40384	2834	N.A.
BT C	14234	13435	6432	**5.61332**
BT D	25657	24854	4564	**3.12975**
BT E	N.A.	92430	2534	N.A.
BT F	N.A.	63990	2234	N.A.

Table 7: IP-LP prediction accuracy after 10 plots

IP PREDICTION ACCURACY (m)				
	IP-std	IP	LP	%improv.
BT A	21766	14163	2197	**34.93063**
BT B	21354	16231	2201	**23.99082**
BT C	6235	6334	2383	**-1.58781**
BT D	9324	9456	2467	**-1.4157**
BT E	36725	20453	1239	**44.30769**
BT F	72843	30495	2034	**58.13599**

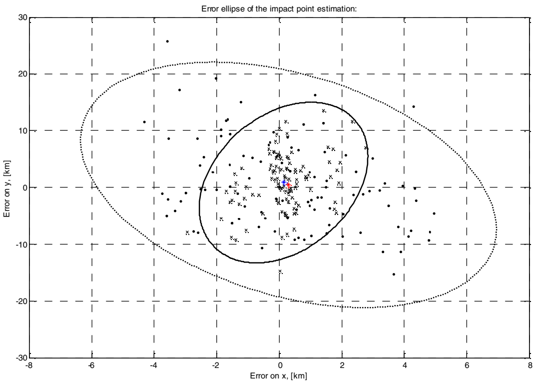

Figure 2: IP prediction for BT A after 10 plots

V. CONCLUSIONS

A new recipe for BT classification and IP-LP prediction has been proposed. It is based upon the exploiting of a-priori information stored on a database and on a multiple model MLE approach. Numerical simulation over a variety of study cases have proven the following:

- the proposed method has good classification capabilities, even with few (5) plots, thus being capable of providing advanced performances within early warning systems;
- the boost phase measurements have been correctly managed, and have been exploited to optimize and, above all, speed up (in the sense that it can start earlier) the IP and LP prediction activities;
- to correctly exploit this last point, burn out time has to be accurately estimated; therefore, a dedicated estimation technique has been proposed, which guarantees enough precision for the whole system to work correctly;
- the IPP performances have been compared with the ones of the MLE proposed in [3], showing significant improvements under certain critical conditions (i.e. long range BT).

The following aspects, in our opinion, merit further investigations:

- the proposed MM-MLE approach manages the classification and IP-LP prediction problems; to complete the whole BT issue, the following activities have to be considered: track initiation, track-detection association, false track management. We believe that the MLE approach can still be the best way to face this whole set of problems;
- The advantages offered by MM-MLE have to be traded off with the high computational load. An algorithm optimization and a deep convergence analysis of the defined functional is mandatory.

VI. REFERENCES

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