Comparison of recursive and batch processing for impact point prediction of ballistic targets

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ABSTRACT.

The paper deals with the problem of impact point prediction of Ballistic Targets (BT) by processing measurements acquired either by 3D surveillance or multifunctional phased-array radars. It is assumed that the radar acquires a limited number of measurements that do not encompass the whole target trajectory; thus the established target track has to be extrapolated ahead in time in order to predict the coordinates of the impact point. In this paper we compare performance of batch (i.e. Maximum Likelihood Estimation, MLE) and recursive (Extended Kalman Filter EKF and Unscented Kalman Filter UKF) filtering techniques.

1 INTRODUCTION

Aim of this paper is to compare performance of batch (i.e. MLE) and recursive (Extended Kalman Filter EKF [1]-[6] and Unscented Kalman Filter UKF [2]-[6]) filtering techniques in predicting the impact point of a BT. To achieve the goal, a kinematic model of the BT have been firstly developed to obtain an accurate mathematical description of the main forces acting on the BT [7]. Once that the model is built, it is possible to conceive proper ballistic target tracking filters and the impact point estimators. It will be shown that, if an a-priori knowledge of the BT physical parameters is available (i.e. the ballistic coefficient), then the MLE batch estimators gives remarkable better performance with respect to the recursive filters. The main problem in the application of the impact point estimators is the lack of knowledge of the BT specific parameters. For the recursive filters and estimators, the problem have been solved [7,8] with the application of a multiple model approach. The recursive filters have been integrated in a Interacting Multiple Model (IMM) [3] tracking architecture for the simultaneous estimate of BT ballistic coefficient and the target velocity and position. Once that the track is established, the BT parameters are estimated and the impact point prediction is performed via the BT equations of motion. As described below, the batch processing is developed for a more accurate prediction of the impact point; our proposal is to estimate the physical characteristics of the BT via a recursive IMM tracking procedure and to adopt an MLE estimator, which uses the IMM outputs and ballistic coefficient estimate, for the impact point prediction. The benefits of this approach are described in this paper which is organised as follows: section 2 briefly recalls the BT model and introduces the radar measurements while section 3 presents the derivation of the MLE estimator. Section 4 contains the description of the EKF, UKF and IMM trackers, section 5 the scenarios definition and the results achieved with the various recursive and batch procedures. Section 6 is dedicated to conclusions and follow on and section 7 lists a number of references.

2 SIMPLIFIED MODELS OF BT KINEMATIC AND RADAR MEASURES

2.1 BT kinematics

Three main forces affect the BT motion: thrust, drag and gravity [6-8]. For the sake of this paper, it is assumed that the BT is in the cruise phase during the BT state vector estimation while drag and gravity are acting on the target body during the re-entry phase. The drag acceleration expression is [6-8]

$$\mathbf{a}_{drag}=\begin{bmatrix} a_{dragx} \\ a_{dragy} \\ a_{dragz} \end{bmatrix} = -\frac{1}{2}\frac{\rho(z)\cdot g_0}{\beta}\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}\cdot\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \quad (2.1)$$

where β is the ballistic coefficient (N/m^2), $\rho(z)$ is the air density function of the height:

$$\rho(z) = 1.21907 \cdot e^{-z/9146.64} \quad (2.2)$$

$\dot{x}, \dot{y}, \dot{z}$ are the velocity components of the BT along the three axes of a Cartesian reference system. The gravity acceleration is considered constant, g_0=9.8m/s^{2}and directed along the z-axis.

2.2 Radar measurements

The measurements, collected by the radar for target tracking, are the range r, elevation ε and azimuth θ; the radar is located at: x_R=0, y_R=0, z_R=0 in all the numerical evaluations that will be done in the paper. The error standard deviations of these measurements are denoted as σ_r (for range), σ_θ (for azimuth) and σ_ε (for elevation). Radar measurements are transformed to the Cartesian coordinates so that the measurement equation is linear

$$\mathbf{z}_k = \mathbf{H}\mathbf{s}_k + \mathbf{v}_k \quad (2.3)$$

where $\mathbf{v}_k$ is the noise on the measured Cartesian co-ordinates; it is zero-mean white Gaussian with covariance matrix $\boldsymbol{R}_k$. For all practical purposes this is a good approximation which greatly simplifies the tracking algorithm; otherwise one would also have to take into consideration the non linearity of the measurement equation.

3 BATCH ESTIMATOR

The proposed technique uses a Maximum Likelihood (ML) approach to the estimation of the Impact Point (IP) of the BT. The aim is to find the state vector components which maximize the likelihood function achieved under the hypothesis of additive and Gaussian measurement error. The probability density function (pdf) of this vector error $\mathbf{e}$ of the radar measurements $R, \vartheta, \varepsilon$ (range, azimuth and elevation respectively) can be written as:

$$P(\mathbf{e}) = \frac{1}{\left[(\pi)^M \left|\mathbf{M}_C\right|\right]^{N/2}} \exp\left[- Tr\left\{\mathbf{M}_C^{-1} \sum_{k=1}^{N} \mathbf{e}_k \mathbf{e}_k^H\right\}\right] \quad (3.1)$$

where $\mathbf{M}_C$ is the covariance matrix, M=3 is its dimension and Tr{**A**}and |**A**| stand for the trace and the determinant of the matrix **A** respectively. Assuming the measurements errors statistically independent, $\mathbf{M}_C$ is a diagonal matrix:

$$\mathbf{M}_C = \begin{bmatrix} 2\sigma_R^2 & 0 & 0 \\ 0 & 2\sigma_\theta^2 & 0 \\ 0 & 0 & 2\sigma_\varepsilon^2 \end{bmatrix} \quad (3.2)$$

Note that in this case the MLE reduces to the classical Least Square Estimation (LSE), that is the minimization of the exponent of the Gaussian pdf or equivalently of its logarithmic version. Therefore the estimated state vector $\hat{\mathbf{x}}$ can be written as:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \lambda(\mathbf{x}) \quad (3.3)$$

where

$$\lambda(\mathbf{x}) = N\left[\ln\left(\sigma_R\sqrt{2\pi}\right) + \ln\left(\sigma_\theta\sqrt{2\pi}\right) + \ln\left(\sigma_\varepsilon\sqrt{2\pi}\right)\right] + \sum_{k=1}^{N}\left\{\frac{[R(\mathrm{k}) - h(\mathbf{x},k)]^2}{2\sigma_R^2} + \frac{[\theta(\mathrm{k}) - f(\mathbf{x},k)]^2}{2\sigma_\theta^2} + \frac{[\varepsilon(\mathrm{k}) - l(\mathbf{x},k)]^2}{2\sigma_\varepsilon^2}\right\} \quad (3.4)$$

where N is the number of radar measurements, $\mathbf{x} = [x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z}]^T$ is the state vector, k identifies the plot acquisition time and:

$$\begin{cases} h(\mathbf{x},k) = \sqrt{x(t_k)^2 + y(t_k)^2 + z(t_k)^2} \\ f(\mathbf{x},k) = arcsin\dfrac{z(t_k)}{\sqrt{x(t_k)^2 + y(t_k)^2 + z(t_k)^2}} \\ l(\mathbf{x},k) = arctg\dfrac{x(t_k)}{y(t_k)} \end{cases} \quad (3.5)$$

Note that the state vector evolution is obtained exploiting the described BT model assuming the ballistic coefficient β known or estimated.

4 RECURSIVE FILTERS AND ESTIMATORS

4.1 EKF theory

This section describes the procedure to derive the EKF starting from the equations describing the forces acting on a BT. For sake of simplicity, the hypothesis of flat earth is done and all the mathematical details connected to the change of co-ordinate system are not reported here. The theory of EKF is widely detailed in [3]. The EKF is used to account for the non linear target state equation due to the presence of drag [6]. At the *k*-th time instant the state vector $\mathbf{s}_k$ contains the position, the speed and acceleration components of target with respect to the Cartesian axes and the ballistic coefficient:

$$\mathrm{s}_k = [x_k \quad \dot{x}_k \quad y_k \quad \dot{y}_k \quad z_k \quad \dot{z}_k \quad \beta_k]^T \quad (4.1)$$

The evolution of the state in time is [7]:

$$\mathbf{s}_{k+1} = \mathbf{\Phi}\mathbf{s}_\mathbf{k} + \mathbf{G}\cdot\left[\mathbf{a}_g + \mathbf{a}_{drag}\right] + \mathbf{w}_k = \mathbf{\Phi}\mathbf{s}_k + \mathbf{G}\cdot\mathbf{a}_g + \mathbf{f}_k(\mathbf{s}_k) + \mathbf{w}_k \quad (4.2)$$

where $\mathbf{\Phi}$ is the 7×7 state transition matrix:

$$\mathbf{\Phi} = \begin{bmatrix} 1 & \mathrm{T} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \mathrm{T} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \mathrm{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.3)$$

G is a 7×3 matrix where *T* is the radar scan time:

$$\mathbf{G} = \begin{bmatrix} T^2/2 & T & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & T^2/2 & T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & T^2/2 & T & 0 \end{bmatrix}^T \quad (4.4)$$

The column vectors $\mathbf{a}_g$ and $\mathbf{a}_{drag}$ contain respectively the gravity and the drag acceleration components along the axes *x*, *y* and *z*:

$$\mathbf{a}_g = \begin{bmatrix} a_{gx} \\ a_{gy} \\ a_{gz} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g_0 \end{bmatrix} \quad (4.5)$$

$$\mathbf{a}_{drag} = \begin{bmatrix} a_{drag\,x} \\ a_{drag\,y} \\ a_{drag\,z} \end{bmatrix} = -\frac{1}{2}\cdot\frac{\rho(z)\cdot g_0}{\beta}\cdot\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\cdot\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \mathbf{f}_k(\mathbf{s}_k) = -\frac{1}{2}\cdot\frac{\rho(s_{k5})\cdot g_0}{s_{k8}}\cdot\sqrt{s_{k2}{}^2 + s_{k4}{}^2 + s_{k6}{}^2}\cdot\begin{bmatrix} s_{k2} \\ s_{k4} \\ s_{k6} \end{bmatrix} \quad (4.6)$$

$\mathbf{f}_k(\mathbf{s}_k)$ is the non linear function which embeds the drag contribution; this function depends on the state components. $\mathbf{w}_k$ is the process noise: it has Gaussian pdf, with zero mean and non singular covariance matrix:

$$\mathbf{Q} = q\begin{pmatrix} \boldsymbol{\psi} & 0 & 0 & 0 \\ 0 & \boldsymbol{\psi} & 0 & 0 \\ 0 & 0 & \boldsymbol{\psi} & 0 \\ 0 & 0 & 0 & \sigma_\beta^2 \end{pmatrix} \quad (4.7)$$

with

$$\boldsymbol{\psi} = \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} \quad (4.8)$$

where q is a scalar quantity accounting for the uncertainty on the target model; the variance σ_β^2 expresses the uncertainty on on the ballistic coefficient. The range of values assumed by β is wide: from about 4000 to about 400000 and it has a relevant impact on the trajectory shape. Given the state $\hat{\mathbf{s}}_{k/k}$ estimated at the *k*-th time instant given all the measurements up to time instant k, with the corresponding estimation covariance matrix $\mathbf{P}_{k/k}$, the prediction at time instant k+1 is:

$$\hat{\mathbf{s}}_{k+1/k} = \boldsymbol{\Phi}\mathbf{s}_k + \mathbf{G}\cdot\left[\mathbf{a}_g + \mathbf{f}_k(\mathbf{s}_k)\right] \quad (4.9)$$

and the covariance matrix of the predicted state is:

$$\mathbf{P}_{k+1/k} = (\boldsymbol{\Phi} + \mathbf{G}\cdot\mathbf{J}_k)\mathbf{P}_{k/k}(\boldsymbol{\Phi} + \mathbf{G}\cdot\mathbf{J}_k)^T + \mathbf{Q} \quad (4.10)$$

where $\mathbf{J}_k$ (3×8 matrix) is the sum of Jacobian of the non linear functions $\mathbf{f}_k(\mathbf{s}_k)$ and $\mathbf{g}_k(\mathbf{s}_k)$ calculated at the state $\hat{\mathbf{s}}_{k/k}$ estimated at the previous step. The Jacobian is:

$$\mathbf{J}_k = \mathbf{F}_k + \mathbf{G}_k = [\nabla_{\mathbf{s}_k}\mathbf{f}_k^T(\mathbf{s}_k)]^T \quad (4.11)$$

For sake of brevity, the Jacobian of above equation is not detailed here.

4.2 UKF Theory

Same as the EKF, the UKF is a recursive MMSE (Minimum Mean Square Error) estimator. But unlike the EKF, the UKF [2] does not approximate the non linear state and measurement equations. It uses the true non linear model of state and/or measurements equation but approximates the pdf of the state vector. This density is still Gaussian, but is specified by a set of deterministically chosen sample (or *sigma*) points. The sigma points completely capture the true mean and covariance of the Gaussian density and when propagated through the non linear system, capture the posterior mean and covariance accurately to the second order for any non linearity [2].

The unscented transform (UT) is a method for calculating the statistics of a random vector which undergoes a non linear transformation. Let $\mathbf{x}$ be the n_x dimensional random vector, $\mathbf{g}: \mathrm{R}^{n_x} \to \mathrm{R}^{n_y}$ a non linear function and $\mathbf{y} = \mathbf{g}(\mathbf{x})$. Assume the mean and the covariance of $\mathbf{x}$ are $\bar{\mathbf{x}}$ and $\mathbf{P}_x$ respectively. The simple procedure for the calculation of the first two moments of $\mathbf{y}$ using the UT is as follows[6].

1. Compute $2n_x$ sigma points χ_i and their weights W_i

$$\chi_i = \bar{\mathbf{x}} + \left(\sqrt{n_x\mathbf{P}_x}\right)_i \quad (4.12)$$

$$i = 1, \ldots, n_x$$

$$W_i = 1/(2n_x) \quad (4.13)$$

$$\chi_i = \bar{\mathbf{x}} - \left(\sqrt{n_x\mathbf{P}_x}\right)_i \quad (4.14)$$

$$i = n_x + 1, \ldots, 2n_x$$

$$W_i = 1/(2n_x) \quad (4.15)$$

where $\left(\sqrt{(n_x\mathbf{P}_x}\right)_i$ is the i-th row or column of the matrix square root of $n_x\mathbf{P}_x$. The weights are normalised (i.e. add up to 1).

2. Propagate each sigma point through the non linear function

$$y_i = \mathbf{g}(\chi_i) \quad i = 0, \ldots 2n_x \quad (4.16)$$

Estimated mean and covariance of $\mathbf{y}$ are computed as

$$\bar{\mathbf{y}} = \sum_{i=1}^{2n_x} W_i y_i$$

$$\mathbf{P}_y = \sum_{i=1}^{2n_x} W_i (y_i - \bar{\mathbf{y}})(y_i - \bar{\mathbf{y}})^T \quad (4.17)$$

Next we describe the implementation of the UKF assuming that at time K the state estimate and its covariance are $\mathbf{s}_{k|k}$ and $\mathbf{P}_{k|k}$ respectively.

- Compute sigma points $\boldsymbol{\xi}_{k|k}(i)$ and weights W_i ($i = 1, \ldots, 12$) corresponding to $\mathbf{s}_{k|k}$ and $\mathbf{P}_{k|k}$;
- Propagate sigma points using state equation (described in the previous section) as follows

$$\boldsymbol{\xi}_{k+1|k}(i) = \Phi_k\left[\boldsymbol{\xi}_{k|k}(i)\right] + \mathbf{G}\begin{pmatrix} 0 \\ -g_0 \end{pmatrix} \quad (4.18)$$

- Compute the mean and covariance of the predicted state $\mathbf{s}_{k+1|k}$ and $\mathbf{P}_{k+1|k}$ using predicted sigma points $\boldsymbol{\xi}_{k+1|k}(i)$, weights W_i as follows

$$\hat{\mathbf{s}}_{k+1|k} = \sum_{i=0}^{8} W_i \boldsymbol{\xi}_{k+1|k}(i)$$

$$\mathbf{P}_{k+1|k} = \mathbf{Q} + \sum_{i=0}^{8} W_i\left[\boldsymbol{\xi}_{k+1|k}(i) - \hat{\mathbf{s}}_{k+1|k}\right]\cdot\left[\boldsymbol{\xi}_{k+1|k}(i) - \hat{\mathbf{s}}_{k+1|k}\right]^T \quad (4.19)$$

- Predict measurements derived from sigma points, that is

$$\varsigma_{k+1|k}(i) = \mathbf{H}\xi_{k+1|k}(i) \quad (4.20)$$

- Predict measurement and covariances

$$\hat{\mathbf{z}}_{k+1|k} = \sum_{i=0}^{8} W_i \varsigma_{k+1|k}(i)$$
$$\mathbf{P}_{zz} = \mathbf{R}_{k+1} + \sum_{i=0}^{8} W_i \left[\varsigma_{k+1|k}(i) - \hat{\mathbf{z}}_{k+1|k}\right] \cdot \left[\varsigma_{k+1|k}(i) - \hat{\mathbf{z}}_{k+1|k}\right]^T \quad (4.21)$$
$$\mathbf{P}_{sz} = \sum_{i=0}^{8} W_i \left[\xi_{k+1|k}(i) - \hat{\mathbf{s}}_{k+1|k}\right] \cdot \left[\varsigma_{k+1|k}(i) - \hat{\mathbf{z}}_{k+1|k}\right]^T$$

where $\mathbf{P}_{zz}$,$\mathbf{P}_{sz}$ are, respectively, the covariance matrix of the measurement and the cross-covariance of the measurement and the state variable.

- Compute the UKF gain and update state and covariance

$$\mathbf{K}_{k+1} = \mathbf{P}_{sz}\mathbf{P}_{zz}^{-1}$$
$$\mathbf{s}_{k+1|k+1} = \mathbf{s}_{k+1|k} + \mathbf{K}_{k+1}\left(\mathbf{z}_{k+1} - \hat{\mathbf{z}}_{k+1|k}\right) \quad (4.22)$$
$$\mathbf{P}_{k+1|k+1} = \mathbf{P}_{k+1|k} - \mathbf{K}_{k+1}\mathbf{P}_{zz}\mathbf{K}_{k+1}^T$$

4.3 Why the IMM architecture

The theory and the application of the IMM have been the subject of many publications, see for instance [3], [7] and [8]. The rationale for choosing the IMM approach for the tracking of a potential BT is essentially due to the fact that the BT characteristics are not generally “a priori” known, thus it is required to “on-line” estimate the BT parameters to maximise the tracker accuracy. The IMM offers the possibility of mixing the output of different filters designed for different BTs, each one having the possibility of adapting its parameters to the target to be tracked, thus permitting the correct tracking of BTs pertaining to different classes. In addition to this, the probability of selecting one of the filters existing in the bank of the IMM gives a clear indication of the confidence of the tracker on the type of target under analysis; this is an intrinsic capability of non co-operative target classification, available “for free” by the IMM. To take into account a β wide range (from about 4000 to about 400000) a 3 filter bank with 3 different initial β values is used in the next simulations.

4.4 CRLB theory applied to recursive processing

Having said that one of the ingredients of the impact point predictor is the recursive filtering based on EKF or UKF, the purpose of this section is to demonstrate the capability of such filters of achieving a satisfactory BT tracking. This will be done by comparing the tracking architecture performance with the corresponding Cramer-Rao Lower Bound (CLRB).

The CRLB, which depends on the measurement model, sensor characteristics and the target state model, plays an important role in algorithm evaluation and assessment of the level of approximation introduced by a specific tracking filter. For a general non linear filtering problem, the optimal recursive state estimator (the tracker) in the Bayesian sense requires the complete posterior density of the state to be determined as a function of time. In the special case of linear/Gaussian estimation, the required density is Gaussian and the solution is the well known Kalman filter, which gives also the accuracy of the estimated track. In the general non linear/non-Gaussian case the problem is very difficult and has no analytic closed form solution. A theoretical formula has been found in [5]; for sake of simplicity, the simulations described in this paper refer to the case of radar detection probability equal to 1. The above mentioned reference gives the strategy to include in the CRLB computation a detection probability <1.

The findings of [5] allow the CRLB computation for the BT tracking accuracy by means of the following methodology:

(i) compute the trajectory for the BT using the equations of sections 2 and 4 process noise is assumed to be absent;
(ii) derive the EKF as described in section 4.1; suppose the a-priori knowledge of the BT parameters (drag coefficient as an example);
(iii) run the EKF of step (ii) using the “nominal” trajectory of step (i) considering the radar measurement errors only in the definition of the measurement errors covariance matrix. In other word, no Monte Carlo trials are required being the effect of the radar uncertainty considered in the computation of the Kalman gain matrix.

The tracker accuracies found with the above mentioned procedures are the CRLB for the BT trajectory under analysis. Once that the CRLB is obtained, it is possible to compare the performance of any other tracking filter; the comparison with the EKF and UKF that we have developed is the subject of next figure 1.

The study case selected for the CRLB-EKF-UKF comparison concerns a simulated BT with following characteristics: single stage, linear consumption of propellant vs. flight time, drag coefficient=80000 N/m^2, radar detection range=300 km, target radar cross section=1 m^2, flight time 500 seconds. The parameters of a notional radar are: range accuracy=50m, azimuth accuracy=0.2°, elevation accuracy =0.2°, data rate=12 seconds P_d=1. Ten ballistic radar plots are used for the study case of figure 1.

The accuracy (3σ values) in the prediction of the BT impact point (BT-IP) is selected for comparison purpose. The BT-IP is computed using the BT dynamic equations defined in section 2 starting from the state vector **s** (see eq. 4.1) computed by the CRLB or by the recursive filters. Figure 1 reports the tracking performances for the CRLB, the EKF and the UKF in terms of the cubic root of uncertainty ellipsoid volume as a function of time.

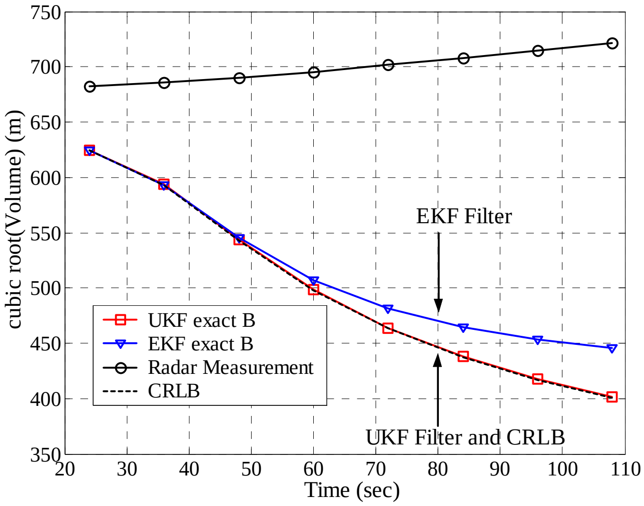

Figure 1: Cubic root of the BT-ellipse of uncertainty volume for the CRLB, the EKF, the UKF and radar measurements.

From figure 1, it is possible to say that the UKF, practically coincident with the CRLB, gives better performance with respect to the EKF. Thus, the computational cost can be one of the driving design factor and it will be discussed in next section 5 which is pertinent to the case of unknown BT parameters.

5 ACHIEVED RESULTS

As discussed in section 4.3, the use of an IMM architecture is recommended due to the lack of knowledge on the characteristics of the BT under tracking. This section has the purposes of (i) presenting the analogous results of figure 1 for the IMM-EKF and the IMM-UKF (see figure 2) and (ii) extending the results of figure 2 by comparing the IMM-EKF and IMM-UKF estimations of the BT-IP with the BT-IP predicted via the MLE procedure described in section 3; figure 3 compares the results in terms of impact point (three sigma) uncertainty ellipse. Given the IMM-EKF as reference, the improvement of MLE and IMM-UKF can be evaluated by comparing the ellipse areas. Table 1 shows the dimension of the ellipse axes and the ratio expressed in percentage among the three areas. The simulated BT and radar parameters are the same used in section 4.4. It must be noted that the MLE requires the knowledge of the BT under tracking (i.e. the drag coefficient), otherwise it fails to converge. Thus, a strategy has to be conceived if the batch processing is applied to achieve the required BT-IP accuracy. For the exercise of figure 3, it is supposed that the estimation of the BT drag coefficient is passed to the MLE by a recursive tracking architecture (IMM-EKF or IMM-UKF) running in parallel with the MLE. A first design consideration is that a scheme like the one of figure 4 has to be implemented to apply the batch processing.

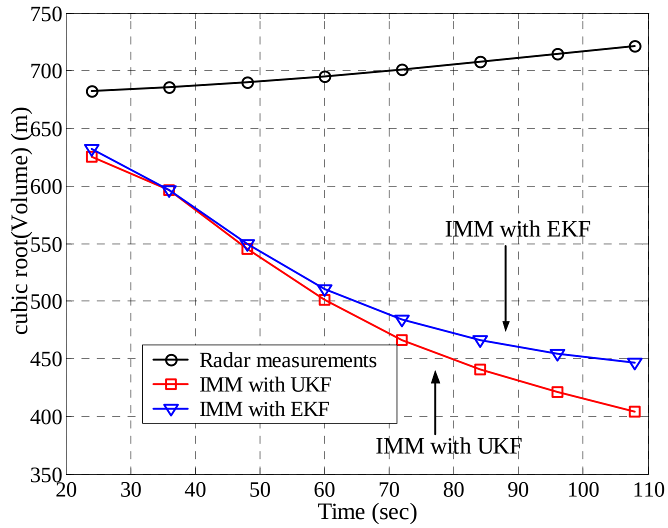

Figure 2: Cubic root of the BT-ellipse of uncertainty volume achieved via the IMM-EKF, the IMM-UKF and radar measurements as a function of time.

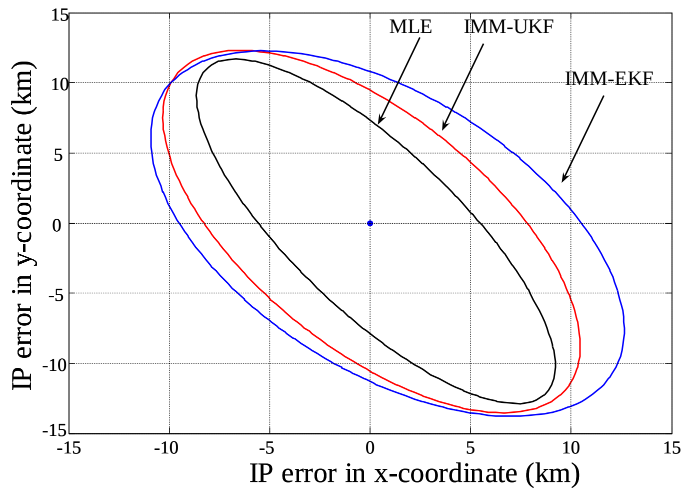

Figure 3: BT-IP ellipse of uncertainty obtained via BT kinematics model starting from state vector estimated by the IMM-EKF, the IMM- UKF and the MLE .

	IMM-EKF	IMM-UKF	MLE
Major axis of uncertainty ellipse (3σ)	31 km	30km	29km
Minor axis of uncertainty ellipse (3σ)	16.6 km	13.5 km	10 km
Δ_{area} of uncertainty ellipse (3σ)	reference	-22%	-43%

Table 1: summary of results presented in figure 3.

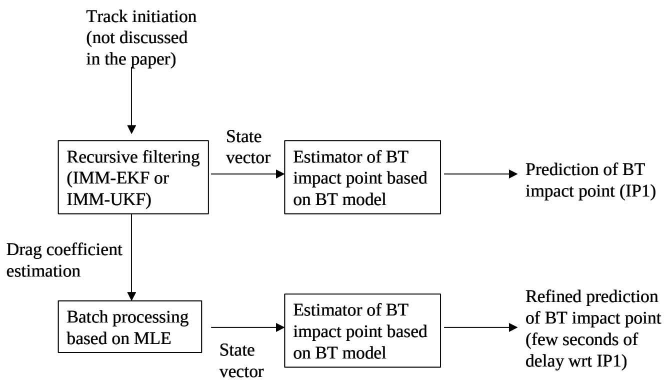

Figure 4: how to implement the batch processing

Thus it seems worthwhile to continue to investigate the use of the MLE notwithstanding the need of a processing scheme like the one of figure 4 and the computational cost. A rough estimation of the computational costs required by the three procedures is the following: said CC the computational cost of the IMM-EKF, the IMM-UKF requires approx. 5CC and the MLE approx. 100CC. Thus, the use of the MLE approach must be carefully considered for the implementation even if it must be taken into account that, usually, the time available for the BT-IP prediction can be in the order of few seconds. In other words, the flight time of BT under analysis is 500 seconds and, roughly, 180 seconds have been lost due to (i) the need of initialising the track (this problem has not been tackled in this paper for simplicity) and (ii) the collection of the five plots required for the "stable" BT-IP prediction. Thus, the state vector **s** (see eqn. 4.1) is available approx. 320 seconds before the BT-IP and it is reasonable to spend 4-5 seconds to improve the IP prediction available from the recursive filters (see figure 4). Such a large amount of time available for the MLE greatly reduces the complexity of the device to be used for the batch processing.

6 CONCLUSIONS

It has been shown that the batch processing based on the MLE approach gives remarkable advantages in predicting the BT-IP with respect to the prediction achievable via recursive filters based on IMM-EKF or IMM-UKF. The recursive filters must be in any case implemented to guarantee the estimation of the parameters that are mandatory to guarantee the MLE convergence. A processing strategy has been presented together with a rough estimation of the computational cost of recursive and batch filtering. A forthcoming paper will describe the results achieved in the attempt of removing the need of the drag coefficient estimation in the batch processing by means of: (i) the use of a more complex likelihood functional with drag coefficient inside (ii) the exploitation of a priori information contained in a database.

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