MAXIMUM LIKELIHOOD ESTIMATOR APPROACH TO DETERMINE THE TARGET ANGULAR CO-ORDINATES IN PRESENCE OF MAIN BEAM INTERFERENCE: APPLICATION TO LIVE DATA ACQUIRED WITH A MICROWAVE PHASED ARRAY RADAR

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SUMMARY

This paper illustrates the application of the Maximum Likelihood Estimation (MLE) technique to determine the Target Direction of Arrival (TDoA) in presence of Main Beam directional Interference (MBI). Application to target plus noise-like interference live data are also considered. MLE technique, which is the best in terms of achievable performance, is a possible candidate as an alternative to the more conventional monopulse usually implemented in multifunctional and surveillance radar.

Multifunctional radars are required to detect, locate and track targets in presence of natural (clutter) and intentional RF directional interference. The interference might impinge on the side lobes as well as on the main beam of the antenna pattern. A possible definition of side lobes (SLI) and main beam (MBI) or skirt (SKI) interference is given in figure 1 where Δθ and Δϕ are suitably specified.

Concerning the location capabilities of the radar, the requirements can be summarized as follows:

- the estimation of the Target Direction of Arrival (TDoA) has to be unbiased,
- the accuracy of the angular co-ordinates has to be a suitable fraction of the -3dB beamwidth depending on the target and the interference conditions.

The paper contains the derivation of an algorithm to estimate the TDoA, via the MLE technique, against side lobes as well as MBI. The purpose is achieved by collecting the signals captured by L high gain beams $\left(B_1(\theta,\phi), B_2(\theta,\phi), \cdots, B_L(\theta,\phi)\right)$ and N low gain auxiliaries $\left(A_1(\theta,\phi), A_2(\theta,\phi), \cdots, A_N(\theta,\phi)\right)$. The processing scheme is illustrated in figure 2.

To test the validity of the MLE approach, data have been recorded in the recent years with a ground based microwave phased array test-bed radar equipped with the conventional three high gain beams: the sum, the difference in azimuth and the difference in elevation. The experimental set-up is described and the achieved TDoA estimation in presence of a MBI is presented; in addition to this, the estimation of the target power and Doppler frequency is also presented.

1. PROBLEM DEFINITION

In this section we formulate the problem of estimating the two target angular co-ordinates (θ_T, ϕ_T) in presence of one or more directional interferences by applying the MLE theory to the data received by a set of the high and low gain beams. The interference may be a mixture of main beam, skirt and side lobe interferences.

Firstly we need to write the probability density function (PDF) of the received signals:

$$\mathbf{z} = \begin{bmatrix} z_{B_1} & z_{B_2} & \cdots & z_{B_L} & z_{A_1} & z_{A_2} & \cdots & z_{A_N} \end{bmatrix}^T \quad (1)$$

where z_{B_i} $i \in \{1,2,\cdots,L\}$ is the sample signal received by the high gain beam B_i, z_{A_k} $k \in \{1,2,\cdots,N\}$ is the one received by the are by the auxiliary low gain beam A_k.

Define the following steering vector whose elements are the values of the patterns of the antenna:

$$\mathbf{v}(\theta,\varphi) \equiv \begin{bmatrix} B_1(\theta,\varphi) \\ B_2(\theta,\varphi) \\ \vdots \\ B_L(\theta,\varphi) \\ A_1(\theta,\varphi) \\ A_2(\theta,\varphi) \\ \vdots \\ A_N(\theta,\varphi) \end{bmatrix} \quad (2)$$

where θ and ϕ are the azimuth and elevation angle.

The vector **z** can be written as the sum of the vector whose elements are the target echo weighted by the L+N beams and of a disturbance vector **d** depending on the directional interference and the receiver noise (it is hypothesized that the clutter has been cancelled):

$$\mathbf{z} = b_T \mathbf{v}^T(\theta_T, \phi_T) + \mathbf{d} \quad (3)$$

where b_T is the complex amplitude of the target that is a deterministic a-priori unknown value; b_T is said to be a nuisance parameter with respect to the θ_T, and ϕ_T (i.e.: the target angular co-ordinates) which are the parameters of interest to estimate.

Hypothesize that the pdf of **d** is Gaussian with zero mean and covariance matrix $\mathbf{M}_d$:

$$p(\mathbf{d}) \approx \mathrm{N}(\mathbf{0}, \mathbf{M}_d) = \frac{1}{\pi^{L+N}|\mathbf{M}_d|}\exp\left[-\mathbf{d}^H\mathbf{M}_d^{-1}\mathbf{d}\right] \quad (4)$$

The expression of the interference covariance matrix is:

$$\mathbf{M}_d = \sum_{i=1}^{M} INR_i \cdot \mathbf{v}(\theta_{I_i}, \phi_{I_i}) \cdot \mathbf{v}^H(\theta_{I_i}, \phi_{I_i}) + \mathbf{N} \quad (5)$$

where $(\theta_{I_i}, \phi_{I_i})$ are the angular co-ordinates and $INR_i = \frac{P_{I_i}}{\sigma_n^2}$ is the interference-to-noise power ratio of the i^th^ interference. **N** is the (L+N)-dimensional identity matrix (we have been assumed for simplicity, but without lack of generality, that the noise power has unity value).

The pdf of **z** in presence of target (H1 hypothesis with the binary detection problem):

$$\begin{aligned} &p(\mathbf{z}) \approx \mathrm{N}(b_T \cdot \mathbf{v}(\theta_T, \phi_T), \mathbf{M}_d) = \\ &\frac{1}{\pi^{L+N}|\mathbf{M}_d|}\exp\left[-[\mathbf{z} - b_T\mathbf{v}(\theta_T,\phi_T)]^H \mathbf{M}_d^{-1}[\mathbf{z} - b_T\mathbf{v}(\theta_T,\phi_T)]\right] \end{aligned} \quad (6)$$

Owing to the Gaussian nature of the *p*(**z**), the MLE problem can be reduced to the following minimization problem:

$$\begin{aligned} &\left(\hat{b}_T, \hat{\theta}_T, \hat{\phi}_T\right) = \\ &\arg\min_{(b,\theta,\phi)}\left\{[\mathbf{z} - b_T\mathbf{v}(\theta,\phi)]^H \mathbf{M}_d^{-1}[\mathbf{z} - b_T\mathbf{v}(\theta,\phi)]\right\} = \\ &\arg\min_{(b,\theta,\phi)} F(b,\theta,\phi) \end{aligned} \quad (7)$$

The minimization of the functional with respect to *b* brings to the following well known matched whitening filter:

$$\hat{b}_T(\theta,\phi) = \frac{\mathbf{v}(\theta,\phi)^H \mathbf{M}_d^{-1}\mathbf{z}}{\mathbf{v}(\theta,\phi)^H \mathbf{M}_d^{-1}\mathbf{v}(\theta,\phi)}. \quad (8)$$

Substituting $\hat{b}_T(\theta,\phi)$ in the equation the functional to minimize becomes:

$$Q(\theta,\phi) = \mathbf{z}^H\mathbf{M}_d^{-1}\mathbf{z} - \frac{\left|\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-1}\mathbf{z}\right|^2}{\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-1}\mathbf{v}(\theta,\phi)} \quad (9)$$

which reduces to either the minimization of (because the term $\mathbf{z}^H\mathbf{M}_d^{-1}\mathbf{z}$ does not depend on θ,ϕ):

$$Q_1(\theta,\phi) = -\frac{\left|\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-1}\mathbf{z}\right|^2}{\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-1}\mathbf{v}(\theta,\phi)} \quad (10)$$

or the maximization of:

$$Q_2(\theta,\phi) = \frac{\left|\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-1}\mathbf{z}\right|^2}{\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-1}\mathbf{v}(\theta,\phi)} \quad (11)$$

One possible intuitive interpretations of the MLE estimator is the following: form a grid of whitening matched filters with different values of the angles (θ,ϕ) and choose the one which gives the maximum value of the signal-to-interference plus noise power ratio at the output.

Another interpretation requires few more mathematics. Rewrite (11) as follows:

$$\begin{aligned} Q_2(\theta,\phi) &= \frac{\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-1}\mathbf{z}\mathbf{z}^H\mathbf{M}_d^{-1}\mathbf{v}(\theta,\phi)}{\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-1}\mathbf{v}(\theta,\phi)} = \\ &\frac{\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-0.5}\mathbf{M}_d^{-0.5}\mathbf{M}_z\mathbf{M}_d^{-0.5}\mathbf{M}_d^{-0.5}\mathbf{v}(\theta,\phi)}{\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-0.5}\mathbf{M}_d^{-0.5}\mathbf{v}(\theta,\phi)} \end{aligned} \quad (12)$$

with the substitutions:

$$\begin{aligned} &\mathbf{M}_z = \mathbf{z}\mathbf{z}^H \\ &\mathbf{v}_w = \mathbf{M}_d^{-0.5}\mathbf{v}(\theta,\phi) \\ &\mathbf{M}_{z,w} = \mathbf{M}_d^{-0.5}\mathbf{M}_z\mathbf{M}_d^{-0.5} \end{aligned} \quad (13)$$

the equation can be written as:

$$Q_2(\theta,\phi) = \frac{\mathbf{v}_w^H(\theta,\phi)\mathbf{M}_{z,w}\mathbf{v}_w(\theta,\phi)}{\mathbf{v}_w^H(\theta,\phi)\mathbf{v}_w(\theta,\phi)} \quad (14)$$

In the above equation, $\mathbf{v}_w$ is the normalized whitened array response and $\mathbf{M}_{z,w}$ is the whitened source plus disturbance covariance matrix. Separating the source and the disturbance terms, we obtain:

$$\begin{aligned} Q_2(\theta,\phi) &= \frac{\mathbf{v}_w^H(\theta,\phi)\mathbf{v}_w(\theta,\phi)}{\mathbf{v}_w^H(\theta,\phi)\mathbf{v}_w(\theta,\phi)} + \\ &SNR\frac{\mathbf{v}_w^H(\theta,\phi)\mathbf{e}_w\mathbf{e}_w^H\mathbf{v}_w(\theta,\phi)}{\mathbf{v}_w^H(\theta,\phi)\mathbf{v}_w(\theta,\phi)} \end{aligned} \quad (15)$$

where $\mathbf{e}_w$ is the whitened array response vector of the desired source. An estimate of the vector $\mathbf{e}_w$ can be obtained by the data by taking the principal eigenvector $\hat{\mathbf{e}}_w$ of $\mathbf{M}_{z,w}$. The maximization of eq. (11) can be approximated as follows:

$$\begin{aligned} &Q_2(\theta,\phi) = \arg\max_{(\theta,\phi)}\frac{\mathbf{v}_w^H(\theta,\phi)\mathbf{e}_w\mathbf{e}_w^H\mathbf{v}_w(\theta,\phi)}{\mathbf{v}_w^H(\theta,\phi)\mathbf{v}_w(\theta,\phi)} = \\ &\arg\max_{(\alpha,\theta,\phi)}\left\|\alpha\mathbf{v}_w(\theta,\phi) - \hat{\mathbf{e}}_w\right\|^2 \end{aligned} \quad (16)$$

In this last form all functions of angles (θ,ϕ) from the denominator have been eliminated by introducing a nuisance parameter α. This expression defines a test that minimizes the error between the appropriately scaled whitened array-response vector $\hat{\mathbf{e}}_w$ and the estimated whitened source array-response vector $\mathbf{v}_w$. The solution to (7) can be considered as a generalization of the monopulse technique; in fact, the monopulse technique can be thought of as finding the angle (θ,ϕ) that maximizes (11) when the whitening matrix $\mathbf{M}_d^{-0.5}$

is the identity matrix, and thus: $\mathbf{v}_w = \mathbf{v}$ and, for a single pulse, $\hat{\mathbf{e}}_w = \mathbf{z}$. The monopulse technique projects the vector $\alpha\mathbf{v}_w - \hat{\mathbf{e}}_w$ onto the subspace spanned by the steering vector of the assumed source direction, and the partial derivatives of that vector with respect to the desired source parameters (θ,ϕ). This technique generates (K+1)-dimensional subspace, where K is the number of desired source parameters (K=2 in our case). The projected error is then driven to zero by choosing the correct α and (θ,ϕ) values. In the case of no interference, the equation giving the parameters $\left(\hat{\theta}_T, \hat{\phi}_T\right)$ is the familiar monopulse ratio expression.

One last interpretation of the MLE is the following. The angular value of the maximum of (11) is found as the maximum of the "so called" radar scan pattern:

$$Q_2(\theta,\phi) = \left|\mathbf{w}^H(\theta,\phi)\cdot\mathbf{z}\right| \tag{17}$$

where

$$\mathbf{w}(\theta,\phi) = \left[\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-1}\mathbf{v}(\theta,\phi)\right]^{-0.5}\mathbf{M}_d^{-1}\mathbf{v}(\theta,\phi) \tag{18}$$

is an adaptive weight vector where (θ,ϕ) scans the look directions of the antenna and the adaptation has to be performed for each look direction. The weight **w** is the well known optimum weighting which has, however, a direction-dependent normalisation factor $\left[\mathbf{v}^H(\theta,\phi)\mathbf{M}_d^{-1}\mathbf{v}(\theta,\phi)\right]^{-0.5}$.

2. *TEST ON RECORDED LIVE DATA*

To test the validity of the MLE approach, several sets of data have been recorded in a site with a ground based test bed microwave Phased Array Radar (PAR) equipped with three high gain beams and a multi channel digital data recorder (MDDR) to acquire simultaneously the live data from three receiving channels. The three receiving channels allowed the data recording from the sum, difference in elevation and difference in azimuth beams. After a brief description of the experimental set-up, the subsequent items are discussed:

- firstly a comparison between the beams reconstructed by simulation and the PAR real ones is performed;
- secondly a suitable phase compensation on the reconstructed beams is estimated to make them congruent with the real ones; then the estimates of the target co-ordinates, obtained after such compensation, are compared to the uncompensated ones;
- an estimate of the interference position has been also performed using the same MLE algorithm, and the estimates with and without phase compensation are compared; the congruence of the reconstructed scenario obtained by the compensated estimates has been analyzed and compared to the target and interference real positions.

2.1. Data capture experimental set up

A series of trials was performed to evaluate the performance of the PAR under various scenarios. A set of suitably positioned interference emulators and transponders were used to produce the required scenario and the antenna was pointed either electronically or manually in the direction of the required interference emitter. Figure 3 depicts the environmental set up.

Two transponders (TP1 and TP2) emulated the useful targets; three interference emulators (IE1, IE2 and IE3) were also present to test the performance of the radar in various conditions to achieve the following goals:

- characterization of radar receiving channels,cancellation of two contemporaneous continuous Noise-Like Interference (NLI),
- one NLI impinging on the main antenna beam,
- cancellation of two contemporaneous continuous NLI with angular distance either less than the radar beamwidth or wider,
- detection of target in presence of two contemporaneous continuous NLIs,
- blanking probability in presence of Coherent Repeater Interference (CRI) only,
- blanking of CRI and cancellation of NLI,
- contemporaneous cancellation of jamming and clutter via STAP algorithms,
- MLE estimation of TDoA under main beam interference conditions.

Live data were acquired soon after the A/D (Analogue-to-Digital) converters by means of the MDDR permitting the simultaneous recording on the three receiving channels. Several Gigabytes of data have been successfully collected.

2.2. Comparison between the theoretical and real radar beams

A discrepancy between the phase of the "difference and sum PAR beams ratio" that has been simulated and the real PAR one has been found during the data recordings. If not properly compensated for, this mismatch could negatively affect the accuracy of the angular estimate based on the MLE algorithm described in the previous section 1. The algorithm can correctly estimate the target angular co-ordinates from real data provided that it uses a good approximation of beam patterns from which the data were acquired.

In Figures 4 and 5 the phase diagrams of the ratio between the simulated difference in azimuth and sum beams and of the ratio between the simulated difference in elevation and sum beam are shown, respectively for the 0° elevation and the 0° azimuth planes. These plots are independent, with a good approximation, respectively of the considered elevation and azimuth angles; they can be considered valid also for the other constant elevation and azimuth planes.

In Figure 6, the phase of the ratio between the difference in azimuth and the sum beams is 107.5° for the positive values of the azimuth, while it is –72.5° for the negative ones. In Figure 7, the phase of the ratio between the difference in elevation and the sum beams is –90.8° for the positive values of the elevation, and it is 89.6° for the negative ones. The phase step is in both cases 180°, as the theory requires.

A data file has been considered for a first evaluation of the phases of the ratio between the difference and sum PAR beams. In this file, the signals received by the sum, the difference in azimuth and the difference in elevation beams of the PAR have been recorded. The scenario included a target (simulated by the transponder TP2 in Figure 3) and an MBI (simulated by the transmitting antenna IE1 in Figure 3). The sweeps corresponding to the 1^{st} and 19^{th} PRT (Pulse Repetition Time) with about one thousand range samples each have been considered.

To calculate the phases of the ratio between the difference and the sum beams, the samples from the 600^{th} range cell to the 700^{th} range cell (in which only the interference is present) have been considered, for both the 1^{st} and the 19^{th} PRTs . The results are shown in Figures 6 to 9 (the phases are module 2π).

The phases are not constant, but they have some oscillations due to slight variation of the INR from sample to sample. Furthermore, it can been noted that the value of the phase of the ratio between the signal received on the difference in azimuth channel and the signal on the sum sensibly changes passing from the 1^{st} to the 19^{th} PRTs (from about –150° to 30°, with a variation of 180°); this can happen if the interference has passed through the null of the difference in azimuth beam during the antenna rotation. The phase of the ratio between the signal received by the difference in azimuth channel and the signal received by the sum, therefore, has a 180° step after the interference has passed through the null, as the theory requires. The phase of the ratio between the signal received by the difference in elevation channel and the signal received by the sum, instead, remains practically constant, except for a soft ripple due to the noise.

Concluding, from the results obtained by the recordings it appears evident that there is some difference between the phase of the simulated beams and the phase of the PAR beams. This fact could deteriorate the angle estimation accuracy unless a suitable phase match between the simulated beams (the ones exploited by the MLE algorithm) and the PAR beams on the two difference channels is made.

2.3. MLE on recorded live data

Using the figures 6÷9, the phase of the ratio between the difference in azimuth and sum beams has been estimated equal to -150°; it has been supposed that the interference had positive azimuth co-ordinate during the considered recording; thus, a phase shift of –260° has been provided to the reconstructed difference in azimuth beam for matching it to the PAR one. Moreover, the phase of the ratio between the difference in elevation and sum beams in the PAR case has been estimated equal to -70°; it has been supposed that the interference had positive elevation during the considered recording, thus a phase shift of +20° has been provided to the reconstructed difference in elevation beam.

Figures 10 and 11 portray the achieved results obtained with the phase compensation. The TDoA estimations obtained with the phase compensation are reliable and favorably compare with the real target position. In fact the antenna was rotating in azimuth clockwise with a constant elevation angle. The limited amount of recorded live data did not allow us to calculate the mean and standard deviation of the TDoA estimate. This analysis will be done in following experimental activities.

3. CONCLUSIONS

An MLE estimator of the target angular co-ordinates has been proposed having the capability of giving satisfactory performance in presence of main beam and sidelobe interference condition. The estimator has been tested using live target plus interference data acquired from a microwave phased array radar. The test has been performed using three high gain beams (sum and the two difference beams). An experimental campaign for the live data acquisition with more advanced radar system and real flying aircraft is foreseen for the next future.

4. REFERENCES

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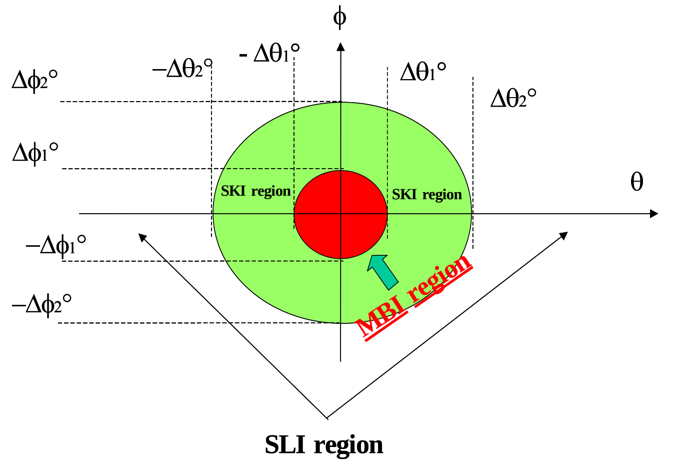

Figure 1. Interference taxonomy definition.

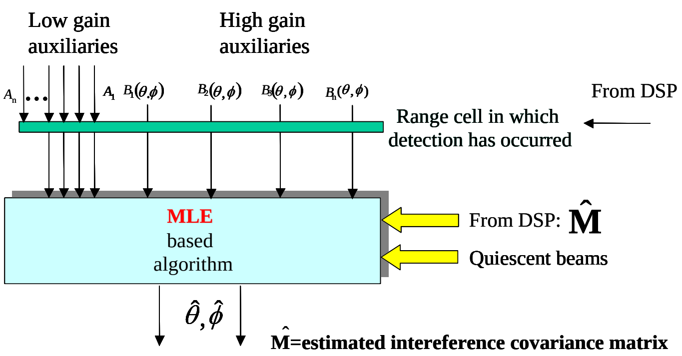

Figure 2. Scheme of the system to estimate the TDoA in presence of side lobes as well as main beam interferences.

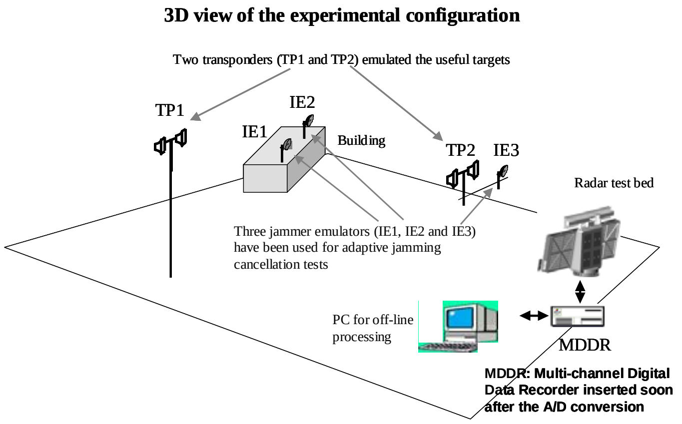

Figure 3. Experimental set up.

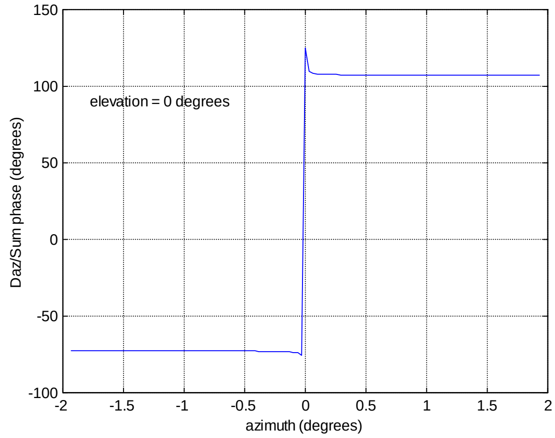

Figure 4. Phase diagram of the simulated ratio between the difference in azimuth and sum beams vs. the azimuth angle. The elevation angle is 0°.

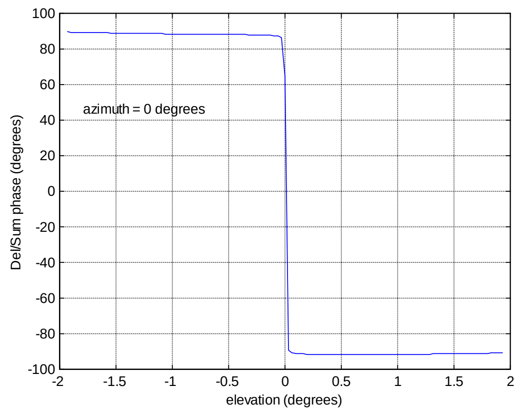

Figure 5. Phase diagram of the ratio between the simulated difference in elevation and sum beams vs. the elevation angle. The azimuth angle is 0°.

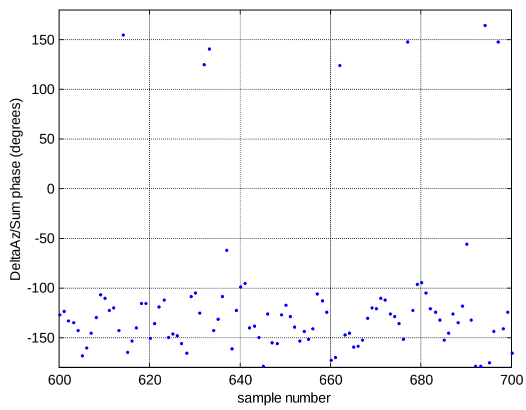

Figure 6. Phase of the ratio between the difference in azimuth and the sum beams; 1st PRT with about one thousand range samples.

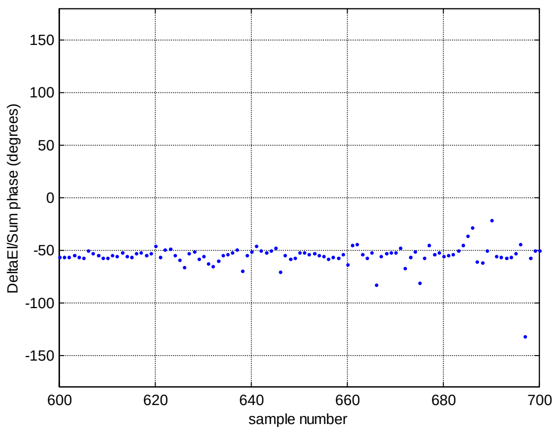

Figure 7. Phase of the ratio between the difference in elevation and the sum beams; 1st PRT with about one thousand range samples.

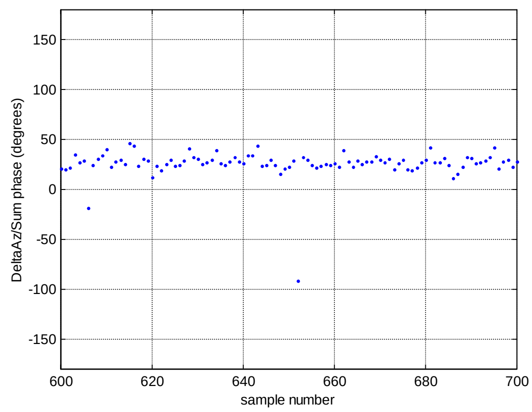

Figure 8. Phase of the ratio between the difference in azimuth and the sum beams; 19th PRT with about one thousand range samples.

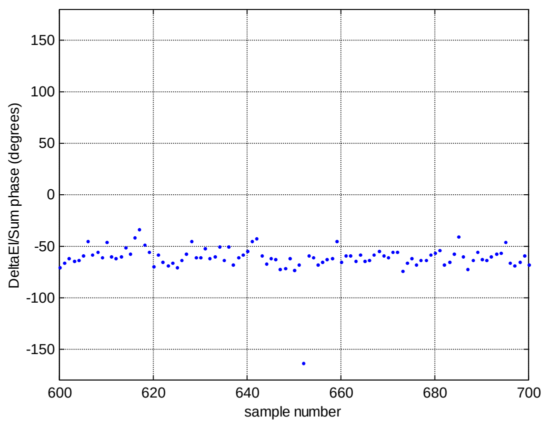

Figure 9. Phase of the ratio between the difference in elevation and the sum beams; 19th PRT with about one thousand range samples.

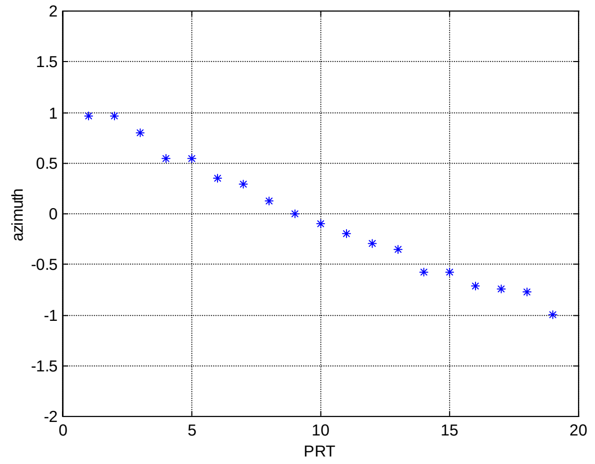

Figure 10. Estimate of the target azimuth; PRT with about one thousand range samples, phase compensation.

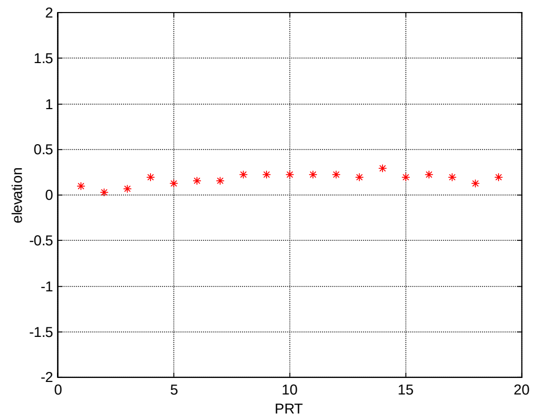

Figure 11. Estimate of the target elevation; PRT with about one thousand range samples, phase compensation.